

# Statistical Fluctuations as Probes of Dense Matter

Berndt Müller<sup>a</sup> \*

<sup>a</sup>Department of Physics, Duke University, Durham, NC 27705, USA

The use of statistical fluctuations as probes of the microscopic dynamics of hot and dense hadronic matter is reviewed. Critical fluctuations near the critical point of QCD matter are predicted to enhance fluctuations in pionic observables. Chemical fluctuations, especially those of locally conserved quantum numbers, such as electric charge and baryon number, can probe the nature of the carriers of these quantum numbers in the dense medium.

## 1. INTRODUCTION

Thermal models for the production of final-state hadrons in nuclear collisions have been extremely successful over a wide range of collision energies ranging from the Bevalac/SIS, over the AGS and SPS, to the RHIC [1–5]. The parametric dependences of the temperature  $T$  and baryon chemical potential  $\mu$  on the nucleon-nucleon center-of-mass energy  $\sqrt{s}$  map out two lines in the phase diagram, which are usually called the *chemical* and *thermal freeze-out* lines. The chemical freeze-out line ( $T_{\text{ch}}, \mu_{\text{ch}}$ ) is deduced from the abundance ratios of different species of hadrons, while the thermal freeze-out line ( $T_{\text{th}}, \mu_{\text{th}}$ ) is deduced from the slopes of the transverse momentum spectra of the emitted hadrons.

What do these results tell us about QCD except that the hadronic matter at freeze-out is quite well described by a state of thermal and chemical equilibrium? Firstly, rather strong arguments have been presented that the chemical and thermal equilibrium can only be established by some complex “prehadronic” dynamics, which requires the presence of gluons as dynamical degrees of freedom [6]. Secondly, the thermal equilibrium description requires the assumption of a strong transverse collective flow of the matter at freeze-out, which exhibits an increasing degree of azimuthal anisotropy as the CM energy increases. This effect, called elliptic flow [7], indicates the presence of a strong transverse pressure very soon (1 fm/c) after the begin of the nuclear reaction [8].

As the chemical freeze-out parameters at SPS and RHIC energies are very close to the expected phase boundary between hadronic matter and the quark-gluon plasma, it is reasonable to conjecture that the produced matter was first thermalized in the deconfined phase and then evolved through the phase boundary into a thermal gas of hadrons. The crucial question is whether any signatures remain in hadronic observables that carry information about this evolution prior to the final freeze-out.

As I will explain, there are reasons to believe that statistical fluctuations [9] in certain hadronic observables may preserve information about earlier times, even though their av-

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erages only tell us about the freeze-out conditions. It is here where the finite volume and the finite life-time of the hadronic fireball produced in relativistic heavy-ion collisions turn out to be advantages rather than deficiencies. The relative size of fluctuations in thermodynamic observables, as compared to the average values, decreases inversely with the volume  $V$  of the system. And if the system would evolve infinitely slowly, the fluctuations at freeze-out would only reflect the freeze-out conditions.

The talk is organized as follows: After a brief review of statistical fluctuations in parts of an equilibrated system, I will discuss fluctuations in the momentum spectra of emitted particles, and later fluctuations in particle abundances, or chemical fluctuations, where the main emphasis will be on the fluctuations of conserved quantum numbers. The talk will conclude with some experimentally relevant considerations and an outlook.

## 2. STATISTICAL FLUCTUATIONS IN A SUBVOLUME

Consider small subsystem of a large system in thermodynamic equilibrium at temperature  $T = \beta^{-1}$ . Let us ask how the average energy  $\langle E \rangle_V$  contained in the subvolume  $V$  changes as a function of temperature. A straightforward calculation yields:

$$\frac{\partial}{\partial \beta} \langle E \rangle_V = \frac{\partial}{\partial \beta} \frac{\text{Tr}_V(H e^{-\beta H})}{\text{Tr}_V(e^{-\beta H})} = -\langle E^2 \rangle_V + \langle E \rangle_V^2. \quad (1)$$

As the right-hand side is just the fluctuation of the energy contained in  $V$ , we have obtained the important result, that the energy fluctuations are determined by the heat capacity of the matter in the subvolume:

$$\langle \Delta E^2 \rangle_V = T^2 \frac{\partial \langle E \rangle_V}{\partial T} = T^2 C_V \quad (2)$$

where  $C_V$  is the heat capacity. Fluctuations of other (conserved) quantities  $\mathcal{O}$  can be calculated in a similar way, by considering the variation of the grand canonical average  $\mathcal{O}_{V,T}$  with respect to a change in the chemical potential  $\mu$  associated with  $\mathcal{O}$ :

$$\langle \Delta \mathcal{O}^2 \rangle_V = T \frac{\partial \langle \mathcal{O} \rangle_{V,T}}{\partial \mu} \equiv T \chi_{\mathcal{O}}. \quad (3)$$

The expression on the right-hand side, apart from the factor  $T$ , is called the *susceptibility*  $\chi_{\mathcal{O}}$ , which measures the response of the medium to a change in the chemical potential associated with the observable  $\mathcal{O}$ . An important example is the magnetic susceptibility  $\chi_M$ , where the observable is given by the magnetization  $M$ , and the magnetic field  $H$  assumes the role of the chemical potential. Examples of interest in the case of hadronic matter are the chiral susceptibility  $\chi_m$  and the Polyakov loop susceptibility  $\chi_L$ , which measure the response of the medium to a change in the quark mass, and to the addition of a free heavy quark, respectively.

## 3. THERMAL FLUCTUATIONS

As outlined above, fluctuations of the thermal energy in a given subvolume are a measure of the change of the average thermal energy with temperature, i.e. the heat capacity

of the matter contained in the subvolume. A similar relation connects the fluctuations of the local temperature to the heat capacity  $C_V$ :

$$\frac{\langle \Delta T^2 \rangle}{T^2} = \frac{1}{C_V}. \quad (4)$$

This relation could be used to measure the specific heat of the hadronic matter created in high-energy nuclear collisions [10,11]. The idea here is to use the slope of the transverse momentum spectrum of emitted particles as a measure of the temperature – assuming that the matter is thermalized – and to look for fluctuations in the event ensemble. Near a critical point, the specific heat diverges, causing the thermal fluctuations to vanish. A pronounced decrease in the observed temperature fluctuations between otherwise identical events would, therefore, indicate an approach to the critical point in the phase diagram.

The simplest way to identify temperature fluctuations is to look for fluctuations in the mean transverse momentum  $\langle p_T \rangle$ . Since the increase of  $C_V$  near  $T_c$  is caused by the increased fluctuations of the modes associated with the order parameter – in the case of the chiral phase transition, the quark condensate  $\langle \bar{q}q \rangle$  – it should be possible to enhance the signal by considering event-by-event fluctuations of observables most sensitive to these modes. For hadronic matter, this would be the mean transverse momentum of low- $p_T$  pions, which are thought to partially arise from the decay of local excitations of the iso-singlet order parameter, the  $\sigma$ -meson mode.

For practical purposes, two observables have been defined [12]:

$$\Phi_p = \left( \frac{\langle \Delta P_T^2 \rangle}{\langle N \rangle} \right)^{1/2} - \left( \overline{\Delta p_T^2} \right)^{1/2} \quad (5)$$

where  $P_T^2$  is the total squared transverse momentum of all particles emitted in an event,  $N$  is the event multiplicity,  $\langle \dots \rangle$  denotes the event average, and  $\overline{\Delta p_T^2}$  the average for the inclusive single particle distribution; and [13]:

$$F_p = \frac{\langle N \rangle \langle \Delta P_T^2 / N \rangle}{\overline{\Delta p_T^2}}. \quad (6)$$

Being a ratio rather than a difference,  $F_p$  may be less sensitive to collective flow effects than  $\Phi_p$ . An estimate for the expected size of  $F$  due to the fluctuations in the  $\sigma$ -field has can be obtained in the framework of the linear sigma model [13]:

$$F_\sigma - 1 \approx 0.14 \left( \frac{\xi_\sigma}{6 \text{ fm}} \right)^2, \quad (7)$$

where  $\xi_\sigma$  is the correlation length of the fluctuations.

This prediction has to be modified for two reasons. First of all, the hadronic system does not decay into free hadrons right at the critical point, even if it reaches this point during its evolution. Instead of observing the large correlation length predicted to occur at  $(T_c, \mu_c)$  in equilibrium, one would thus expect to observe the reduced correlation length associated with the freeze-out parameters  $(T_f, \mu_f)$ , see Fig. 1. Furthermore, even under conditions of criticality, the correlation length only diverges if the system has enough time to develop long-range fluctuations of the order parameter. The theory of dynamical

critical phenomena yields an equation determining the change in the inverse correlation length  $m(t) = \xi(t)^{-1}$  as the system evolves [14]:

$$\frac{dm}{dt} = -\Gamma(m(t)) (m(t) - m_{\text{eq}}(t)). \quad (8)$$

Here  $m_{\text{eq}}$  is the inverse correlation length in equilibrium – which vanishes at the critical point – and the relaxation rate  $\Gamma(m) \sim m^z$  with some positive exponent  $z$ . As critical conditions are approached, the relaxation time diverges exhibiting the well-known effect of critical “slowing down”.

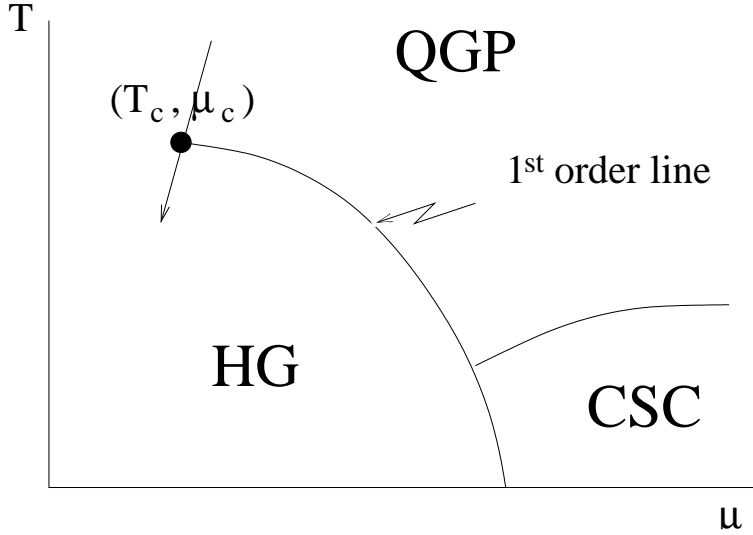


Figure 1. Schematic QCD phase diagram. The first-order transition line separating the quark-gluon plasma (QGP) phase and the hadronic gas (HG) phase terminates in a critical point  $(T_c, \mu_c)$ , where the transition is of second order. Under favorable conditions, the hot system created in a heavy ion collision may transiently pass through the critical point, before it freezes out into hadrons that no longer interact.

If the critical point is reached in a nuclear collision, the system rapidly expands across it and freezes out soon afterwards. According to (8) the correlation length  $\xi(t)$  remains smaller than  $\xi_{\text{eq}}$  as  $T_c$  is approached from above, but then decreases less rapidly than  $\xi_{\text{eq}}$ . The prediction is thus that the effective correlation length at the moment of freeze-out is larger than one would expect under equilibrium conditions, reflecting the temporary proximity to the critical point [14,15].

Experimental data for  $\langle p_T \rangle$  fluctuations have been obtained by NA49 for Pb+Pb collisions at the CERN-SPS [16] and recently by STAR for Au+Au collisions at RHIC [17]. While the NA49 data reflect mean  $p_T$  fluctuations in a forward rapidity region, the STAR data were taken at midrapidity. NA49 reported values of  $F = 1.004 \pm 0.004$  or  $\Phi_p = 0.6 \pm 1$  MeV after corrections for two-particle (HBT) correlations. The value from STAR is  $\Phi_p \approx 35$  MeV.

#### 4. CHEMICAL FLUCTUATIONS

Fluctuations in the chemical composition of the emitted hadron yields can potentially probe the microscopic structure of the emitting matter. An example is the quark number susceptibility

$$\chi_q = T \frac{\partial}{\partial \mu_q} \langle \bar{q} \gamma^0 q \rangle, \quad (9)$$

which measures the response of the net quark density to a change in the quark chemical potential. A different, but related quantity is the chiral susceptibility  $\chi_m = T \partial \langle \bar{q} q \rangle / \partial m$ , which measures the response of the quark condensate to a change in the current quark mass.  $\chi_q$  has an isoscalar and an isovector component

$$\begin{aligned} \chi_S &= T^{-1} \langle [\Delta(u^\dagger u) + \Delta(d^\dagger d)]^2 \rangle, \\ \chi_{NS} &= T^{-1} \langle [\Delta(u^\dagger u) - \Delta(d^\dagger d)]^2 \rangle, \end{aligned} \quad (10)$$

which have both been determined on the lattice [18]. The results show that  $\chi_S \approx \chi_{NS}$ , indicating that fluctuations in the u- and d-quark densities are uncorrelated.

As chemical properties are generally carried by particles, chemical fluctuations are determined by fluctuations in the corresponding particle numbers, and their changes are governed by particle transport processes. A weakly coupled hadronic gas (HG) and a perturbative quark-gluon plasma (QGP) differ significantly in this respect, as listed in Table 1. This implies that measurable differences in the fluctuations associated with

Table 1

Quantum numbers of electric charge ( $Q$ ) and baryon number ( $B$ ) of particles and their abundances in a weakly coupled hadron gas (HG) and quark-gluon plasma (QGP).

	$ Q $	particles	fraction		$ B $	particles	fraction
HG	$e$	$\pi, K, \dots$	$\sim 2/3$	HG	1	$N, \bar{N}, \Lambda, \dots$	$\ll 1$
QGP	$e/3, 2e/3$	$q, \bar{q}$	$\sim 1/2$	QGP	$1/3$	$q, \bar{q}$	$\sim 1/2$

charge and baryon number exist between a hadron gas and a quark-gluon plasma.

The second important point is that fluctuations of locally conserved quantities, such as the net electric charge or the net baryon number (or the net strangeness!) cannot be erased by local reactions. They can only be modified due to particle transport over larger distances, i.e. diffusion, and may thus be fixated early, if the evolution of the system is sufficiently rapid [19,20]. Consider the total net electric charge of all particles emitted within a rapidity window  $\Delta y$  in a high-energy nuclear collision. The fluctuation of this quantity  $\langle \Delta Q^2 \rangle_{\Delta y}$  is proportional to the volume associated with emission into this rapidity window, i.e. to  $\Delta y$  itself in a boost invariant scenario. On the other hand, the rate of diffusion of net charge into and out of this volume is independent of the size of the rapidity interval. Therefore, early fluctuations of the net charge contained in a sufficiently large

rapidity interval  $\Delta y \geq \Delta y_{\min}$  may survive. The questions are, how large is  $\Delta y_{\min}$ , and how much do the predictions for a HG and a QGP differ?

We first address the last question. We estimate the fluctuations per unit volume in the two different phases in the weak coupling limit. In the hadron gas we have

$$\begin{aligned}\langle \Delta B^2 \rangle_{\text{HG}} &= N_B + N_{\bar{B}}, \\ \langle \Delta Q^2 \rangle_{\text{HG}} &\approx N_{\pi^+} + N_{\pi^-} + N_{K^+} + N_{K^-} + \dots,\end{aligned}\tag{11}$$

where  $N_i$  denotes the number of hadrons of species  $i$  in the subvolume, and  $N_B$  ( $N_{\bar{B}}$ ) counts all (anti-)baryons. In the quark-gluon plasma we have

$$\begin{aligned}\langle \Delta B^2 \rangle_{\text{QGP}} &= \frac{1}{9}(N_q + N_{\bar{q}}), \\ \langle \Delta Q^2 \rangle_{\text{QGP}} &\approx \frac{4}{9}(N_u + N_{\bar{u}}) + \frac{1}{9}(N_d + N_{\bar{d}} + N_s + N_{\bar{s}}),\end{aligned}\tag{12}$$

with the analogous notation of the quark numbers. Neglecting the contribution from strange quarks, the net charge and baryon number fluctuations in the QGP are related as

$$\frac{\langle \Delta B^2 \rangle_{\text{QGP}}}{\langle \Delta Q^2 \rangle_{\text{QGP}}} = \frac{2}{5}.\tag{13}$$

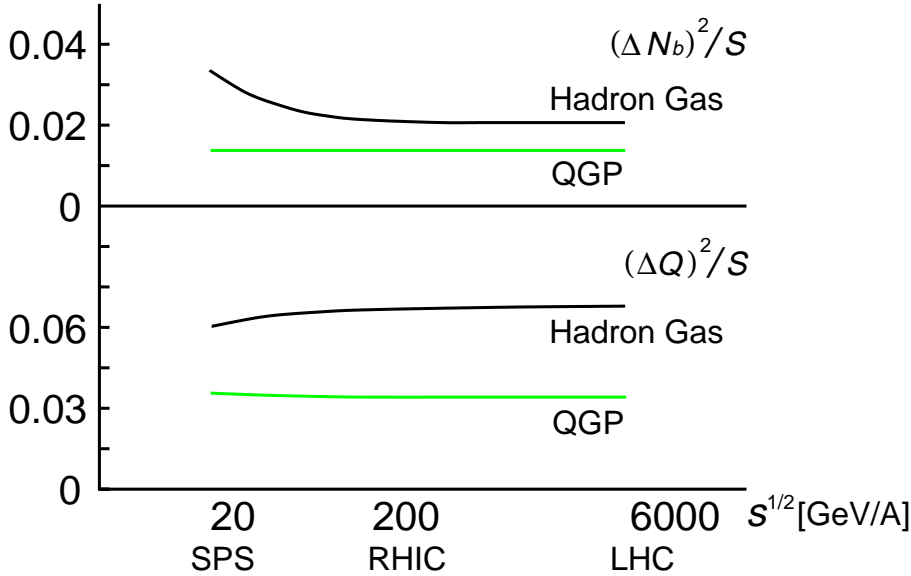


Figure 2. Mean square fluctuations per unit entropy of baryon number and charge for a weakly interacting hadronic gas versus a weakly interacting quark-gluon plasma. The change with beam energy is due to the dependence on baryon chemical potential [19].

Since it is difficult, if not impossible, to measure the size of an observed subvolume accurately, it is desirable to normalize the fluctuations to another extensive quantity.

One possibility is to choose the total entropy  $S$ , which is closely related to the total number of particles in the subvolume. For the QGP one can derive the analytical relation ( $\mu$  denotes the baryon chemical potential)

$$\frac{\langle \Delta B^2 \rangle_{\text{QGP}}}{S_{\text{QGP}}} = \frac{5}{37\pi^2} \left( 1 + \frac{22}{111} \left( \frac{\mu}{\pi T} \right)^2 + \dots \right), \quad (14)$$

which also determines the charge fluctuations by virtue of (13). Quantitative estimates for the hadron gas phase are somewhat more complex, requiring corrections for the decay of higher resonances. The resulting estimates for the net charge and baryon number fluctuations, divided by the entropy, are compared in Fig. 2 with the QGP predictions over an energy range spanning from the SPS to the LHC. The HG prediction is always significantly higher, and at RHIC energies and beyond the net charge fluctuations provide for greater discrimination than the baryon fluctuations. However, we will argue shortly that early net baryon number fluctuations have a greater chance of survival in a narrow rapidity interval.

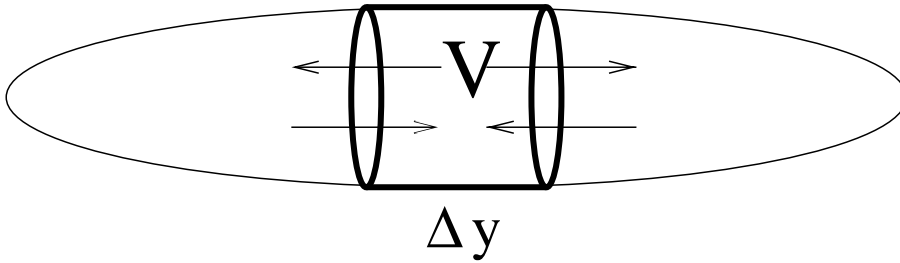


Figure 3. We consider net charge and baryon number fluctuations in a cylindrical volume  $V$ , defined by a rapidity window  $\Delta y$  in the boost invariant Bjorken model. Conservation laws dictate that the total charge and baryon number in  $V$  can only change due to particle transport through the endcaps of  $V$ .

Next we address the question whether the fluctuations generated during a QGP phase can survive the final state expansion in the HG phase. For simplicity, we consider the boost invariant case of a longitudinal expansion introduced by Bjorken, illustrated in Fig. 3. Considering a rapidity interval  $\Delta y = 1$ , we see that the left and right boundary separate from another with the relative velocity  $\Delta v \approx 0.76c$ . The typical longitudinal velocity component of a baryon in the hadronic phase is

$$\bar{v}_z = (8T/\pi M)^{1/2} \approx 0.32c. \quad (15)$$

implying that most baryons will not be able to move out of the rapidity interval, in which they were contained at the moment of hadronization. Thus, we would expect that the baryon number fluctuations generated in a rapidity interval  $\Delta y \geq 1$  during the QGP phase will remain frozen during the HG phase. A more detailed analysis of the transport

of baryons through the left- and right-hand surfaces of a cylindrical volume shows that baryon number fluctuations from the QGP phase can survive if  $\Delta y > \bar{v}_z$  [19].

The time evolution of fluctuations can also be studied in the framework of transport theory [21]. Consider the distribution of particles in rapidity space as a function of proper time  $\tau$ :

$$n(y, \tau) = n_{\text{eq}}(y) + f(y, \tau), \quad (16)$$

where  $f(y, \tau)$  describes the deviation from equilibrium. The relaxation toward equilibrium is described by the Langevin equation

$$\frac{\partial f}{\partial \tau} = -\frac{1}{\tau_{\text{eq}}} f + \gamma \frac{\partial^2 f}{\partial y^2} + \xi, \quad (17)$$

where  $\tau_{\text{eq}}$  is the relaxation time,  $\gamma$  is the coefficient describing diffusion of particles in rapidity, and  $\xi$  is the noise term. For a locally conserved particle density,  $\tau_{\text{eq}}^{-1} = 0$ , and the decay of a fluctuation  $f(y)$  is solely governed by diffusion. Making use of the equivalent Fokker-Planck equation for the Fourier components of the rapidity fluctuations  $f(k, \tau)$  one can show that the Gaussian width  $\sigma_k(\tau)$  of the component  $f(k, \tau)$  satisfies the equation

$$\frac{\partial}{\partial \tau} \sigma_k^2 = -2\gamma k^2 (\sigma_k^2 - \chi_k) \quad (18)$$

where  $\chi_k = \int d\eta e^{ik\eta} \langle f(y+\eta) f(y) \rangle$ . There is no relaxation of the fluctuations toward equilibrium for the  $k = 0$  component due to the conservation law; higher Fourier components decay exponentially with the rate  $2k^2\gamma$ .

The diffusion constant  $\gamma$  is determined by the rapidity transfer in particle collisions and the average time between collisions:  $\gamma = (\delta y_{\text{coll}})^2 / (2\tau_{\text{coll}})$ . With some additional simplifying assumptions one then finds that the mean square fluctuation  $\langle \Delta N^2 \rangle$  in a rapidity interval  $\Delta y$  decays according to

$$\langle \Delta N^2 \rangle = \Delta y \left( \chi_0 + (\sigma_0^2 - \chi_0) G(\Delta y_{\text{diff}} / \Delta y) \right), \quad (19)$$

where  $G(x)$  is a universal function. For large times,  $\langle \Delta N^2 \rangle$  decays slowly toward its equilibrium value.  $\Delta y_{\text{diff}}^2 = 2 \int \gamma(\tau) d\tau$  describes the minimal rapidity interval for which initial state fluctuations survive. Shuryak and Stephanov obtained the estimates [21]

$$\Delta y_{\text{diff}}^{(\pi)} \approx 2.2, \quad \Delta y_{\text{diff}}^{(N)} \approx 0.9 \quad (20)$$

for the rapidity intervals controlling charge and baryon number fluctuations, respectively. However, these estimates are based on collision rates deduced from a hadronic cascade model (RQMD) that does not include a deconfined phase, thus possibly overestimating the size of the required rapidity windows.

## 5. EXPERIMENTAL CONSIDERATIONS AND RESULTS

For the net charge fluctuations, an elegant way to eliminate the size of the observed volume is to consider the fluctuations in the quantity  $R = N_+ / N_-$ . These can be related to the net charge fluctuations by means of the relation [20]

$$D = \langle N_{\text{ch}} \rangle \langle \Delta R^2 \rangle = 4 \langle \Delta Q^2 \rangle / \langle N_{\text{ch}} \rangle. \quad (21)$$



It is useful to correct the quantity  $D$  for two trivial effects by defining a variable  $\tilde{D}$  as [22]

$$D = C_\mu C_y \tilde{D}, \quad (22)$$

with  $C_\mu = \langle N_+ \rangle^2 / \langle N_- \rangle^2$  accounting for the net average charge in the rapidity interval (due to the baryon excess) and  $C_y = 1 - \langle N_{\text{ch}} \rangle_{\Delta y} / N_{\text{ch}}^{(\text{tot})}$  correcting for the effect of global charge conservation. For an uncorrelated thermal pion gas one expects  $\tilde{D} = 4$ . Decays of higher meson resonances (such as  $\omega, \rho, \eta$ ) reduce the value of  $\tilde{D}$  by about 30%. For a weakly interacting QGP one anticipates  $\tilde{D} \approx 1$ .

Preliminary results for  $\tilde{D}$  have been reported by the NA49 collaboration for Pb+Pb collisions at 40, 80, and 158 GeV/u beam energy [16]. A value slightly in excess of  $\tilde{D} = 4$  was found for all three beam energies and independent of the selected pseudorapidity interval  $0.3 < \Delta\eta < 3.5$ . The STAR collaboration has reported first results for net charge fluctuations in Au+Au collisions at  $\sqrt{s_{NN}} = 130$  GeV in the pseudorapidity window  $|\Delta\eta| < 0.7$ . The value obtained by STAR is  $\tilde{D} \approx 3$ , as expected for a nearly baryon-free hadron resonance gas [17].

## 6. SUMMARY AND OUTLOOK

Fluctuations are sensitive probes of the microscopic structure of dense matter. Local temperature fluctuations can indicate the presence of a second-order phase transition or critical point, while net charge and baryon number fluctuations provide information about the particle modes that govern the charge and baryon number transport. In all cases, the rapid expansion of the fireball in its last stages is essential for the survival of fluctuations established at early times.

In addition to thermal fluctuations, the fireball may exhibit fluctuations due to the initial state [23]. These could be probed independently in p+p or p+A collisions. For example, such fluctuations may be probes of the quasiclassical coherent glue fields in a fast-moving nucleus [24]. Similarly, large nonstatistical fluctuations, such as disoriented chiral condensates, can signal the presence of unstable collective modes at some time during the course of the heavy ion collision.

Preliminary data from the NA49 and STAR experiments indicate the presence of both, nontrivial  $p_T$  fluctuations near central rapidity and net charge fluctuations similar to those of a pion or hadron resonance gas. The measurement of net baryon number fluctuations remains a formidable experimental challenge, since they require the detection of neutral as well as charged baryons. However, the case may not be entirely hopeless, as plans by the PHENIX collaboration to measure antineutrons with high efficiency demonstrate [25].

Fluctuations are a rich topic with many theoretical challenges and opportunities. Lattice gauge theory can make quantitative predictions for fluctuations near thermal equilibrium. The wide array of possible fluctuation observables remains largely unexplored. Microscopic models can simulate final state effects on fluctuations and provide useful guidance about which observables contain valuable information. There is ample room for the exploration of novel analysis strategies, as our picture of the space-time evolution of nuclear collisions at RHIC energy comes into focus. One especially promising class of observables are the *balance functions*, which can provide a more differential measure of quantum number fluctuations in phase space [26]. The subject also promises a wealth

of experimental data, as several RHIC detectors, foremost PHOBOS and STAR, are well equipped to explore event-by-event fluctuations in a wide range of observables.

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